Algebra Forum

Phil Daro
Catching Up

• Students with history of going slower are not going to catch up without spending more time and getting more attention.
• Who teaches whom.
• Change the metaphor: not a “gap” but a knowledge debt and need for know-how. The knowledge and know-how needed are concrete, the stepping stones to algebra.
Immediate focus

• Grades 4 through 7 are a critical opportunity
• What is your plan to change the way you invest student and teacher time in grades 4 through 7?
• What additional resources are you adding to the base (time)?
System or Sieve?

• A system of interventions that catch students that need a little help and gives it
• Then catches those that need a little more and gives it
• Then those who need even more and gives it
• By layering interventions, minimize the number who fall through to most expensive
### Intensification

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<th>Needed by S</th>
<th>Intervention</th>
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<tr>
<td>Keeps up</td>
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<td>None</td>
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<td>Designed double period ramp-up course, Extended day, Summer schools <strong>Ramp-Up or Navigator</strong></td>
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</table>
Dylan Wiliam on Instructional Assessment

- **Long-cycle**
  - Span: across units, terms
  - Length: four weeks to one year

- **Medium-cycle**
  - Span: within and between teaching units
  - Length: one to four weeks

- **Short-cycle**
  - Span: within and between lessons
  - Length:
    - day-by-day: 24 to 48 hours
    - minute-by-minute: 5 seconds to 2 hours
### Strategies for increasing instructional assessment (Wiliam)

<table>
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<th>Engineering effective classroom discussions, questions, and learning tasks</th>
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<td>Activating students as the owners of their own learning</td>
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<td>Activating students as instructional resources for one another</td>
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<td>Misconceptions from earlier grades disrupt participation</td>
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<tr>
<td>More than a year behind, misconceptions from many earlier grades</td>
<td>Intensive ramp-up course</td>
</tr>
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</table>
Why do students have to do math. problems?

1. to get answers because Homeland Security needs them, pronto
2. I had to, why shouldn’t they?
3. so they will listen in class
4. to learn mathematics
Why give students problems to solve?

To learn mathematics.

Answers are part of the process, they are not the product.

The product is the student’s mathematical knowledge and know-how.

The ‘correctness’ of answers is also part of the process. Yes, an important part.
Wrong Answers

- Are part of the process, too
- What was the student thinking?
- Was it an error of haste or a stubborn misconception?
Three Responses to a Math Problem

1. Answer getting
2. Making sense of the problem situation
3. Making sense of the mathematics you can learn from working on the problem
Answers are a black hole: hard to escape the pull

- Answer getting short circuits mathematics, making mathematical sense
- Very habituated in US teachers versus Japanese teachers
- Devised methods for slowing down, postponing answer getting
Answer getting vs. learning mathematics

• USA:
  How can I teach my kids to get the answer to this problem?
  Use mathematics they already know. Easy, reliable, works with bottom half, good for classroom management.

• Japanese:
  How can I use this problem to teach mathematics they don’t already know?
Teaching against the test

$3 + 5 = \[ \]$ 
$3 + [ ] = 8$
$[ ] + 5 = 8$

$8 - 3 = 5$
$8 - 5 = 3$
$N$ stands for the number of hours of sleep Ken gets each night. Which of the following represents the number of hours of sleep Ken gets in 1 week?

- $N + 7$
- $N - 7$
- $N \times 7$
- $N + 7$
Anna bought 3 bags of red gumballs and 5 bags of white gumballs. Each bag of gumballs had 7 pieces in it. Which expression could Anna use to find the total number of gumballs she bought?

A \((7 \times 3) + 5 =\)
B \((7 \times 5) + 3 =\)
C \(7 \times (5 + 3) =\)
D \(7 + (5 \times 3) =\)
An input-output table is shown below.

- Input (A) Output (B)
- 7        14
- 12       19
- 20       27

Which of the following could be the rule for the input-output table?

A. $A \times 2 = B$
B. $A + 7 = B$
C. $A \times 5 = B$
D. $A + 8 = B$

Butterfly method

\[
\begin{array}{c}
3 \\
4 \\
\hline
\end{array} + \ \begin{array}{c}
\hline \\
\hline \\
\hline
\end{array} + \begin{array}{c}
3 \\
\end{array}
\]
\[ \frac{9}{3} + \frac{1}{3} + \frac{4}{1} = \frac{13}{12} \]
Use butterflies on this TIMSS item

\[
\frac{1}{2} + \frac{1}{3} + \frac{1}{4} =
\]
Foil FOIL

- Use the distributive property
- It works for trinomials and polynomials in general
- What is a polynomial?
- Sum of products = product of sums
- This IS the distributive property when “a” is a sum
Answer Getting

Getting the answer one way or another and then stopping

Learning a specific method for solving a specific kind of problem (100 kinds a year)
• Wadja get?
• Howdja do it?
• Do you remember how to do these?
• Here is an easy way to remember how to do these
• Should you divide or multiply?
• Oh yeah, this is a proportion problem. Let’s set up a proportion?
Canceling

\[ \frac{x^5}{x^2} = x \cdot x \cdot x \cdot x \cdot x / x \cdot x \]

\[ \frac{x^5}{x^5} = x \cdot x \cdot x \cdot x \cdot x / x \cdot x \cdot x \cdot x \cdot x \]
Word Problem from popular textbook

- The upper Angel Falls, the highest waterfall on Earth, are 750 m higher than Niagara Falls. If each of the falls were 7 m lower, the upper Angel Falls would be 16 times as high as Niagara Falls. How high is each waterfall?
Imagine the Waterfalls: Draw
Diagram it
The Height of Waterfalls
Heights
Height or Waterfalls?

750 m.
Heights we know

750 m.

7 m.
Heights we know and don’t

750 m.

7 m.

7 m.
Heights we know and don’t

Angel = 750 + d + 7
Niagara = d + 7
Same height referred to in 2 ways

16d = 750 + d

Angel = 750 + d + 7
Niagara = d + 7
\[ 16d = 750 + d \]
\[ 15d = 750 \]
\[ d = 50 \]

Angel = 750 +d + 7
Niagara = d + 7

Angel = 750 +50 +7 = 807
Niagara = 50 + 7 = 57
Activate prior knowledge

- What knowledge?
- “Have you ever seen a waterfall?”
- “What does water look like when it falls?”
What is this problem about?
What is this problem about?

HEIGHT!
Delete “waterfalls” and it does change the problem at all. Replace waterfall with flagpoles, buildings, hot air balloons…it doesn’t matter.

The prior knowledge that needs to be activated is knowledge of height.
Bad Advice

• “eliminate irrelevant information”
  – Before you have made sense of the situation, how would you know what is relevant?
  – After you have made sense, you are already past the point of worrying about relevance.
What mathematics do we want students to learn from work on this problem …

• Sasha went 45 miles at 12 mph. How long did it take?
that they can use on this problem?

– Xavier went 85 miles in two and a half hours. Going at the same speed, how long would it take for Xavier to go 12 miles.
Teaching to diagram

• Teaching student to create a diagram about the relationships of the quantities in the problem that helps them create a mental mathematical model of that situation.
• Teach diagramming to one student at a time then to partners then to larger groups
• As a group
Specific techniques:

- What does that phrase mean? (pointing to a phrase that refers to a quantity).
- Play the naïve student who doesn’t understand the situation.
  - This is what Harold did is he right?
- Can you show me in a diagram?
  - Explain your diagram to me
  - Where is (quantity) in your diagram?
  - Can you label you diagram?
- What are the quantities and how are they related?
Water Tank

- We are pouring water into a water tank. 5/6 liter of water is being poured every 2/3 minute.
  - Draw a diagram of this situation
  - Make up a question that makes this a word problem
Test item

• We are pouring water into a water tank. 5/6 liter of water is being poured every 2/3 minute. How many liters of water will have been poured after one minute?
Where are the numbers going to come from?

- Not from water tanks. You can change to gas tanks, swimming pools, or catfish ponds without changing the meaning of the word problem.
Numbers: given, implied or asked about

- The number of liters poured
- The number of minutes spent pouring
- The rate of pouring (which relates liters to minutes)
Diagrams are reasoning tools

• A diagram should show where each of these numbers come from. Show liters and show minutes.

• The diagram should help us reason about the relationship between liters and minutes in this situation.
• The examples range in abstractness. The least abstract is not a good reasoning tool because it fails to show where the numbers come from. The more abstract are easier to reason with, if the student can make sense of them.
Why do students struggle?

- Misconceptions
- Bugs in procedural knowledge
- Mathematics language learning
- Meta-cognitive lapses
- Lack of knowledge (gaps)
- Disposition, belief, and motivation (see AYD)
Misconceptions:

where they come from and how to fix them
Misconceptions about misconceptions

• They weren’t listening when they were told
• They have been getting these kinds of problems wrong from day 1
• They forgot
• The other side in the math wars did this to the students on purpose
More misconceptions about the cause of misconceptions

- In the old days, students didn’t make these mistakes
- They were taught procedures
- They were taught rich problems
- Not enough practice
Maybe

- Teachers’ misconceptions perpetuated to another generation (where did the teachers get the misconceptions? How far back does this go?)
- Mile wide inch deep curriculum causes haste and waste
- Some concepts are hard to learn
Whatever the Cause

• When students reach your class they are not blank slates
• They are full of knowledge
• Their knowledge will be flawed and faulty, half baked and immature; but to them it is knowledge
• This prior knowledge is an asset and an interference to new learning
Second grade

- When you add or subtract, line the numbers up on the right, like this:
  - 23
  - +9

- Not like this:
  - 23
  - +9
Third Grade

- $3.24 + 2.1 = ?$
- If you “Line the numbers up on the right “ like you spent all last year learning, you get this:
  - 3.2 4
  - $+ 2.1$
- You get the wrong answer doing what you learned last year. You don’t know why.
- **Teach: line up decimal point.**
- **Continue developing place value concepts**
Fourth and Fifth Grade

- Time to understand the concept of place value as powers of 10.
- You are lining up the units places, the 10s places, the 100s places, the tenths places, the hundredths places.
Stubborn Misconceptions

- Misconceptions are often prior knowledge applied where it does not work
- To the student, it is not a misconception, it is a concept they learned correctly…
- They don’t know why they are getting the wrong answer
Research on Retention of Learning: Shell Center: Swan et al

Misconception Learning verses Remedial Learning:
Test Scores

Students who were taught by addressing misconceptions
Students who were taught using remedial measures
A whole in the head
A whole in the whose head?

\[
\frac{3}{4} + \frac{1}{3} = \frac{4}{7}
\]
The Unit: one on the Number Line
Between 0 and 1

0  1/4  3/4  1  2  3  4
Adding on the ruler
making sense of math. problems
Our goal is to teach students to make sense of, produce and reason with abstract diagrams that show all the numbers, their relationships.
• A good sense making practice is to first make a more concrete diagram in early sense making, then revise it to a more abstract diagram for reasoning purposes.
• A good teaching practice is to have students compare and discuss different diagrams for the same problem.
Word problems are a “genre” of text

• Read poems differently than we read novels or instructions or a movie review or a recipe for corn fritters
• Genre are contracts between writer and reader: writer makes assumptions about how the reader will read; reader needs to make the right assumptions
• Genre knowledge must be taught and learned
Word problem genre assumptions

• The “context’ of word problems is this:
  – I am reading this for math class
• This is about numbers of [who cares what]
• Focus language effort on:
  – Where are the numbers coming from in this situation? Domains “numbers of”, units
  – Numbers expressed as phrases in text correspond to mathematical phrases (expressions)
  – The verb is “equals” (+,- are conjunctions)
• Can I find or formulate two different phrases (expressions) that refer to the same number?
early stages in making sense of the problem situation

Focus on the domain of meaning from which the numbers come, what are the quantities in the situation?

Imagine the quantities referred to in the problem by words, numbers, letters, phrases.

Represent the relationships among quantities in diagrams, tables, words.

Work out what happens “each time” by increasing \( x \) by 1 and figuring what happens to \( y \); relate each time to the rows of a table and static diagram.
Imagine and define the domain mathematically, what are the sets of numbers referred to by the variables?

Animated understanding: from each time to any time. Generalized grasp of the mathematical relationship in the problem situation expressed as an equation; $y$ for any $x$.

Equations and graphs

Can interpret transformed equations (that express $x$ in terms of $y$, for example) in terms of the problem situation
Making Sense of the mathematics in the problem

- How does the table correspond to the graph? Where is this table row on the graph?
- What does a point (a region) on the graph refer to in the problem situation?
- How does the equation correspond to the situation (what do the letters refer to, what do the operations and the = say about the situation?)?
- How do the equation, graph, table, diagram correspond to each other feature by feature?
- What kind of equation(s) is it?
Make up a word problem for which the following equation is the answer

• \( y = 0.03x + 1 \)
Anna bought 3 bags of red gumballs and 5 bags of white gumballs. Each bag of gumballs had 7 pieces in it. Which expression could Anna use to find the total number of gumballs she bought?

A \( (7 \times 3) + 5 = \)
B \( (7 \times 5) + 3 = \)
C \( 7 \times (5 + 3) = \)
D \( 7 + (5 \times 3) = \)
## Tiered Levels of Intervention

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<th>Intervention Tier</th>
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<td>Tier 1</td>
<td>Classroom Q&amp;A, partner, teacher’s ear &lt;br&gt; <em>Professional development</em></td>
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<td>Not bringing enough from earlier lessons each day</td>
<td>Extra support with regular program</td>
<td></td>
<td>Homework clinic, tutoring, attention beyond regular class &lt;br&gt; <em>Scheduling / targeted use of adopted materials</em></td>
</tr>
<tr>
<td>Misconceptions disrupt participation and success in mathematics (gaps)</td>
<td>In depth concentration on troublesome concepts <em>(not initial teaching)</em></td>
<td>Tier 2</td>
<td>Sustained instruction with special materials beyond regular class period and/ or summer school &lt;br&gt; <em>Navigator</em></td>
</tr>
<tr>
<td>More than a year behind, misconceptions from many years</td>
<td>Extra time and focus on critical mathematics to accelerate to grade level</td>
<td>Tier 3</td>
<td>Designed double period ramp-up course, summer school: &lt;br&gt; <em>Navigator Summer,</em></td>
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</table>
Malcolm Swan example: navigator

- Goldilocks problems that lead to concepts through work on misconceptions (faulty prior knowledge)
- Discussion craftily scaffolded
- Instructional assessment on all cycles, especially within lesson
- Tasks easy as possible to engage as activities that also hook straightaway to questions that lead to concept
- “encouraged uncertainties” at the door of insights
Student Materials, Classroom Routines, and Tasks

About Tasks: Interpreting Multiple Representations

\[
\begin{align*}
\frac{3}{8} & & \frac{1}{4} & & \frac{3}{8} & & \frac{2}{5} \\
\frac{3}{4} & & \frac{1}{8} & & \frac{2}{25}
\end{align*}
\]
Student Materials, Classroom Routines, and Tasks

About Tasks: Making Posters
Social and meta-cognitive skills have to be taught by design

- Meta-cognitive engagement modeled and prompted
  - Does this make sense?
  - What did I do wrong?
- Social skills: learning how to help and be helped with math work => basic skill for algebra: do homework together, study for test together
Marita's homework

2. Marita completed this homework. Correct all the mistakes in her work.

Comparing Decimals

1. Circle the greatest (largest) of the three numbers:
   - 6.4
   - 6.85
   - 6.325

Answer: 6.85 or 6.325

2. Write the following in descending order (greatest to least):
   8.67 8.8 8.09 8.4 8.38 8.675 8.5
   Answer: 8.8 8.5 8.4 8.67 8.38 8.09 8.675
   8.8 8.675 8.67 8.5 8.4 8.38 8.09

3. Circle all of the numbers that are greater than 0.45:
   - 0.15
   - 0.3
   - 0.5
   - 0.625
   - 0.375

4. Circle all of the numbers that are less than 0.75:
   - 0.706
   - 0.6
   - 0.815
   - 0.9
   - 0.085

3. Discuss Marita's work with a partner and try to understand the mistakes that she made. Write advice to Marita on how to correct her mistakes.
### Checkpoint: Adding and Subtracting Fractions

**Checkpoint 1**

For each problem, circle the correct answer. Show your work.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer Options</th>
<th>Correct Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{50}{100} + \frac{6}{100} - \frac{9}{100} )</td>
<td>A 47, B 65, C 64, D 53, E 47</td>
<td>A 47, B 65, C 64, D 53, E 47</td>
</tr>
<tr>
<td>2. ( \frac{15}{17} - \frac{8}{17} + \frac{4}{17} )</td>
<td>A 11, B 12, C 15, D 17, E 10</td>
<td>A 11, B 12, C 15, D 17, E 10</td>
</tr>
<tr>
<td>3. ( 6 - \frac{4}{3} )</td>
<td>A ( \frac{4}{3} ), B ( \frac{2}{3} ), C ( \frac{1}{3} ), D 2, E ( \frac{2}{3} )</td>
<td>A ( \frac{4}{3} ), B ( \frac{2}{3} ), C ( \frac{1}{3} ), D 2, E ( \frac{2}{3} )</td>
</tr>
<tr>
<td>4. ( \frac{1}{3} - \frac{2}{3} )</td>
<td>A ( \frac{1}{5} ), B ( \frac{2}{3} ), C 1, D ( \frac{1}{3} ), E ( \frac{3}{3} )</td>
<td>A ( \frac{1}{5} ), B ( \frac{2}{3} ), C 1, D ( \frac{1}{3} ), E ( \frac{3}{3} )</td>
</tr>
</tbody>
</table>

### Check

- 7. \( \frac{5}{6} - \frac{1}{6} = \)
  - A 1, B 4, C 3, D 2, E 1
- 8. \( \frac{2}{9} + \frac{5}{9} = \)
  - A 10, B 5, C 7, D 7, E 1
- 9. \( \frac{4}{7} + 5 = \)
  - A \( \frac{4}{7} \), B \( \frac{4}{7} \), C \( \frac{4}{7} \), D \( \frac{8}{7} \), E \( \frac{8}{7} \)
- 10. \( 3 - \frac{4}{5} = \)
  - A \( \frac{1}{5} \), B \( \frac{4}{5} \), C \( \frac{1}{5} \), D \( \frac{3}{5} \), E \( \frac{7}{5} \)

Check your answers using the Checkpoint 1 Answer Key on page 16 in the Group Task Book. Then share one problem with the group.
Diagnostic Teaching

- Goal is to surface and make students aware of their misconceptions

- Begin with a problem or activity that surfaces the various ways students may think about the math.

- Engage in reflective discussion (challenging for teachers but research shows that it develops long-term learning)

Problem 2

A _____  B _____  C _____

Measure each line segment above.

a. Write the length of line segment $A$ to the nearest $\frac{1}{4}$ inch.

b. Write the length of line segment $B$ to the nearest $\frac{1}{8}$ inch.

c. Write the length of line segment $C$ to the nearest $\frac{1}{16}$ inch.

d. Josh said, “Wow! I can’t believe those line segments are all different lengths. They all looked the same to me.” Josh is wrong. How could you convince him that all of the line segments have the same length even though their measurements look different?
<table>
<thead>
<tr>
<th>Problem</th>
<th>Diagram and Words</th>
<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Three pizzas are shared equally among six people. How much does each person get?</td>
<td>Diagram</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Words</td>
<td></td>
<td></td>
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</tbody>
</table>
## Operations and Word Problems

<table>
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<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Diagram

- \( \frac{1}{2} \)
- \( \frac{1}{6} \) \( \frac{1}{6} \) \( \frac{1}{6} \)

### Words

How many sixths are in one-half?
5. **Diagram**

\[
\begin{array}{cccccccccccc}
\hline
& & & & & & & & & & & & \\
6 & & & & & & & & & & & & \\
1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\
\hline
\end{array}
\]

**Words**

How many halves are in six?
Work with a partner to solve these problems. You will need scissors.

1. This graph shows how the speed of a race car varies during the second lap of a race:

![Graph showing speed variation](image)

Distance Along Track

Discuss with your partner which of these tracks the car was driving around.

A

B

C

D

E

F

G

Write your reasons for each track you reject.
Expressions and Equations

Work Time

Work with a partner.

Pablo and Jin use two different calculators to find the values of these expressions. But they get different results from the different calculators.

<table>
<thead>
<tr>
<th>Expressions</th>
<th>Pablo’s Results</th>
<th>Jin’s Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 + 4 \cdot 2 =$</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>$8 + 4 \div 2 =$</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>$8 - 4 \cdot 2 =$</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>$8 - 4 \div 2 =$</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

1. Discuss with your partner what you think is going on. What are the two calculators doing differently?
Name: Angela
15. Simplify:

\[5(3m - 2n) - (m - 3n)\]

\[= 5(3m + (-1)2n) + (-1)m + (-1)(-3n)\]

\[= 15m - 2n - m + 3n\]

\[= 15m - m - 2n + 3n\]

\[= (15 - 1)m + (-2 + 3)n\]

\[= 14m + (3 - 2)n\]

\[= 14m + n\]

Name: Mario
15. Simplify:

\[5(3m - 2n) - (m - 3n)\]

\[= 5(3m - 2n) + (-1)(m - 3n)\]

\[= 5(3m - 2n) - m + 3n\]

\[= (5)3m - (5)2n - m + 3n\]

\[= 15m + (-10)n - m + 3n\]

\[= (15 - 1)m + (-10 + 3)n\]

\[= 14m - 13n\]

2. a. What types of errors did Angela make?

b. What types of errors did Mario make?

3. Write some advice to Angela and Mario to help them avoid making these mistakes.
Angela completed the following homework. Unfortunately, the work contains errors.

- Mark whether each problem is correct or incorrect using a colored pen.
- Correct the problems that are wrong.
- Figure out the thinking that caused the errors.

**Name:** Angela

For Problems 1–4, find the next term in the sequence.

1. 4, 9, 14, 19, 24, **48**
2. 4, 7, 10, 13, 16, **32**
3. 4, 8, 12, 16, 20, **24**
4. 2, 8, 18, 32, 50, **100**

5. a. How many dots would be in the 5th and 6th patterns in this sequence?

    ![Dots Pattern]

    **9 and 11**

    b. Write a formula for the sequence.  **n + 2**
Knowing Fractions

Problem 1

Tamika measures a “0 to 1” number line and notices that $\frac{1}{4}$ and $\frac{3}{4}$ are 2 inches apart.

a. How long is the whole number line in inches?

b. Say how you know.
Knowing Fractions

Lesson

Problem 1

Figure A

Figure B

Figure A is divided into the same fractional parts as Figure B.

a. What is that fractional part?

b. Say how you know.
Gabby and Malaya each ate $\frac{1}{6}$ of a pizza but they didn’t eat the same amount of pizza. How can that be?
Knowing Fractions

**Problem 1**

On Tuesday, Malaya spent \( \frac{2}{3} \) of her homework time doing math. She still has \( \frac{1}{2} \) hour of homework left to do.

a. What is the total time Malaya planned for homework?

b. Show your solution on a number line.

[Number line from 0 to 1]

- Write your solution as a complete sentence.
1. Angela completed the following homework.
   a. Mark whether each problem is correct or incorrect using a red pen.
   b. Correct any wrong answers.

   Name: Angela

11. Misha wanted to buy a book that was on sale at 40% off. If the book regularly costs $28, what was the sale price?
    
    \[0.40 \times 28 = 11.20\]  
    The sale price was $11.20

12. Mr. Stevens paid his bill at a restaurant. The cost of the meal for his family was $60. He wanted to add a tip of 15% of the bill.
    a. What was the total cost of the meal and the tip?
       
       \[60 \times 0.15 = 9.00\]  
       The total cost of the bill was $60.90
    
    b. How much was the tip?  
       $0.90
    
    c. How much would a 20% tip be?  
       $1.20
    
    d. How much would a 20% tip be on a bill of $100?  
       $20.00

13. Mrs. Lopez, Marita’s mother, got a loan from the bank to buy a car. She borrowed $3,000. She got a 9% interest rate. At the end of one year, she has to repay the amount she borrowed and 9% of that amount.
    
    What is the total amount she has to repay?  
    $270.00
4 students want to share 1 sandwich equally.

a. Sketch lines to divide this sandwich equally among the students.

b. What part of the sandwich does each person get? Write your answer as a fraction.

c. Write an equation that shows how you divided the sandwiches.
Understanding Fractions

Lesson

Problem 1

On a recent math test, Amir wrote that $\frac{1}{5}$ was equivalent to 0.5. His answer was incorrect.

a. Why was Amir’s response incorrect?

b. How would you help him correct his thinking?