Foundations for Success
National Mathematics Advisory Panel

Report to the
California Algebra Forum
October 14, 2008
President Executive Order
April 2006

• The Panel will advise the President and the Secretary of Education on the best use of scientifically based research to advance the teaching and learning of mathematics, with a specific focus on preparation for and success in algebra.
What Concerns Led to the President’s Order?

• National prosperity and safety in international context
  - Role of mathematics in national well-being
  - *Gathering Storm*
  - Workforce of the future

• Options for individuals and families
  - College admission and graduation
  - Candidacy for technical workforce
  - Earning power
  - Adaptability
Math Proficiency of U.S. Students

- International comparisons
- Low fractions of proficiency on NAEP
- Falling proficiency at higher grades
- Heavy remedial demand upon entry into college
- Achievement gap

*Algebra as a gateway*
Overview

• Task Groups
  - Conceptual Knowledge and Skills
  - Learning Processes
  - Instructional Practices
  - Teachers
  - Assessment

• Subcommittees
  - Standards of Evidence
  - Survey of Algebra Teachers
  - Instructional Materials

• Reports
  - Final Report
  - 8 Task Group and Subcommittee Reports
Basis of the Panel’s work

• Review of 16,000 research studies and related documents.

• Public testimony gathered from 110 individuals.

• Review of written commentary from 160 organizations and individuals

• 12 public meetings held around the country

• Analysis of survey results from 743 Algebra I teachers
Evidence Guidelines

• Executive Order
  - Marshal the “best available scientific evidence.”
  - Review “research relating to proven-effective and evidence-based mathematics instruction.”

• What is the best available scientific evidence?
  - 3 broad categories of quality.
    • Highest quality = high internal and external validity.
    • Promising or suggestive = has limitations.
    • Opinion = values, impressions, or weak evidence.
Two Major Themes

• “First Things First”
  - Positive results can be achieved in a reasonable time at accessible cost by addressing clearly important things now.
  - A consistent, wise, community-wide effort will be required.

• “Learning as We Go Along”
  - In some areas, adequate research does not exist.
  - The community will learn more later on the basis of carefully evaluated practice and research.
  - We should follow a disciplined model of continuous improvement.
Curricular Content

Three Formal Products:

- Major Topics of School Algebra
- Critical Foundations
- Benchmarks
Curricular Content

Streamline the Mathematics Curriculum in Grades PreK-8:

• Follow a Coherent Progression, with Emphasis on Mastery of Key Topics

• Focus on the Critical Foundations for Algebra
  - Proficiency with Whole Numbers
  - Proficiency with Fractions
  - Particular Aspects of Geometry and Measurement

• Avoid Any Approach that Continually Revisits Topics without Closure
Curricular Content

Benchmarks Should Guide:

• Classroom Curricula
• Mathematics Instruction
• Textbook Development
• State Assessment
Curricular Content

The Major Topics of School Algebra

*Covering all of school algebra traditionally extending over two courses, Algebra I and Algebra II*

- Symbols and Expressions
- Linear Equations
- Quadratic Equations
- Functions
- Algebra of Polynomials
- Combinatorics and Finite Probability
Curricular Content

An Authentic Algebra Course

All school districts:

• Should ensure that all prepared students have access to an authentic algebra course, and

• Should prepare more students than at present to enroll in such a course by Grade 8.
Curricular Content

What Mathematics Do Teachers Need to Know?

• For early childhood teachers:
  - Topics on whole numbers, fractions, and the appropriate geometry and measurement topics in the Critical Foundations of Algebra

• For elementary teachers:
  - All topics in the Critical Foundations of Algebra and those topics typically covered in an introductory Algebra course

• For middle school teachers:
  - The Critical Foundations of Algebra
  - All of the Major Topics of School Algebra
Learning Processes

Scientific Knowledge on Learning and Cognition Needs to be Applied to the Classroom to Improve Student Achievement:

• Most children develop considerable knowledge of mathematics before they begin kindergarten.

• Children from families with low incomes, low levels of parental education, and single parents often have less mathematical knowledge when they begin school than do children from more advantaged backgrounds. This tends to hinder their learning for years to come.

• There are promising interventions to improve the mathematical knowledge of these young children before they enter kindergarten.
Learning Processes

• To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, factual knowledge and problem solving skills.

• Limitations in the ability to keep many things in mind (working-memory) can hinder mathematics performance.
  - Practice can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand.
  - Learning is most effective when practice is combined with instruction on related concepts.
  - Conceptual understanding promotes transfer of learning to new problems and better long-term retention.
Learning Processes

Children’s goals and beliefs about learning are related to their mathematics performance.

• Children’s beliefs about the relative importance of effort and ability can be changed.

• Experiential studies have demonstrated that changing children’s beliefs from a focus on ability to a focus on effort increases their engagement in mathematics learning, which in turn improves mathematics outcomes.
Learning Processes

• Engagement and sense of efficacy for Black and Hispanic students can be increased in mathematical learning contexts.

• Teachers and other educational leaders should consistently help students and parents understand that an increased emphasis on the importance of effort is related to improved mathematics grades.
Mathematically Knowledgeable Classroom Teachers Have a Central Role in Mathematics Education.

• Evidence shows that a substantial part of the variability in student achievement gains is due to the teacher.

• Less clear from the evidence is exactly what it is about particular teachers—what they know and do—that makes them more effective.

• The mathematics preparation of elementary and middle school teachers must be strengthened as one means for improving teacher effectiveness in the classroom.
• Currently there are multiple pathways into teaching.
  - Research indicates that differences in teachers’ knowledge and effectiveness between these pathways are small or non-significant compared to very large differences among the performance of teachers within each pathway.

• The Panel recommends that research be conducted on the use of full-time mathematics teachers in elementary schools, often called elementary math specialist teachers.
The Math Panel recommends policy initiatives that put in place and carefully evaluate the effects of:

- Raising base salaries for teachers of mathematics to attract more mathematically qualified teachers into the workforce;
- Salary incentives for teachers of mathematics for working in locations that are difficult to staff; and
- Opportunities for teachers of mathematics to increase their base salaries substantially by demonstrable effectiveness in raising student achievement.
Instructional Practices

Instructional practice should be informed by high quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers.

- All-encompassing recommendations that instruction should be student-centered or teacher-directed are not supported by research.
Instructional Practices

Formative assessment enhances mathematics achievement, particularly when:

- Information is used to determine focus of instruction
- Expert teachers offer advice
- Computer-assisted instruction or peer tutoring is a component
Instructional Practices

Research on students who are low achievers, have difficulties in mathematics, or have learning disabilities related to mathematics tells us that the effective practice includes:

- Explicit methods of instruction available on a regular basis
- Clear problem solving models
- Carefully orchestrated examples/sequences of examples.
- Concrete objects to understand abstract representations and notation.
- Participatory thinking aloud by students and teachers.
Instructional Practices

Use of technology shows promise when:

- Computer-assisted instruction supports drill and practice
- Well designed tutorials are delivered through computer-assisted instruction
- Learning is supported by the careful, targeted application of computer programming

More research is needed
Instructional Practices

A review of 11 studies that met the Panel’s rigorous criteria (only one study less than 20 years old) found limited or no impact of calculators on calculation skills, problem solving, or conceptual development over periods of up to one year.

• This finding is limited to the effect of calculators as used in the 11 studies and the Panel recommends more research.

Mathematically precocious students with sufficient motivation appear to be able to learn mathematics successfully at a much higher rate than normally-paced students, with no harm to their learning.
Instructional Materials

• U. S. mathematics textbooks are far too long -- often 700-1000 pages. Mathematics textbooks are much smaller in many nations with higher mathematics achievement than the U.S. Excessive length makes our books unnecessarily expensive and tends to undermine coherence and focus.

• Publishers must ensure the mathematical accuracy of their materials.
Assessment

• NAEP and state tests must focus on the mathematics that students should learn, with scores reported and tracked over time.

• States and NAEP need to develop better quality control and oversight procedures to ensure that test items:
  - Are of the highest quality.
  - Measure what is intended.
  - Do not include design or wording problems that provide unintended sources of difficulties.
Research Policies and Mechanisms

It is essential to produce methodologically rigorous scientific research in crucial areas of national need, such as the teaching and learning of mathematics.

- More research is needed that identifies:
  - Effective instructional practices, materials, and principles of instructional design,
  - Mechanisms of learning,
  - Ways to enhance teachers’ effectiveness, including teacher education, that are directly tied to objective measures of student achievement, and
  - Item and test features that improve the assessment of mathematical knowledge.
Research Policies and Mechanisms

As in all fields of education, the large quantity of studies gathered in literature searches on important topics in mathematics education is reduced appreciably once contemporary criteria for rigor and generalizability are applied.

- The Panel recommends that governmental agencies that fund research give priority not only to increasing the supply of research that addresses mathematics education, but also to ensuring that such projects meet stringent methodological criteria.
Research Policies and Mechanisms

• Leaders of graduate programs in education and related fields should ensure attention to research design, analysis, and interpretation for teachers and those entering academic and educational leadership positions in order to increase the national capacity to conduct and utilize rigorous research.

• New funding should be provided to establish support mechanisms for career shifts (K, or career, awards from the NIH represent one example). Many accomplished researchers who study the basic components of mathematics learning are not directly engaged in relevant educational research.
Research Policies and Mechanisms

• Support should be provided to encourage the creation of cross-disciplinary research teams, including expertise in educational psychology, sociology, economics, cognitive development, mathematics, and mathematics education.

• PreK-12 schools should be provided with incentives and resources to provide venues for, and encourage collaboration in, educational research.

• Unnecessary barriers to research should be lowered.
A Rare Effort

- Two years of work
- Expert panel plus extensive support from staff and consultants
- Full spectra of philosophies and relevant expertise
- Unprecedented base of examined literature and testimony received
- Millions of dollars invested
- Focus on best available scientific evidence
Important products

• A unanimously adopted final report
• 45 specific, high level findings and recommendations
• Five task group reports
• Three subcommittee reports
• All told, about 800 pages of detailed analysis
American Students Need Effective Teamwork from Us

- Extensive evidence shows that our students achieve at only fair levels when compared internationally
- Many, many need remedial work upon entry to college
- Too few are ready for technical curricula
- They can do better. We can do better.

“A consistent, wise, community-wide effort will be required.”
Advice to the Community

• Focus on a short list of most important steps for the next five years
• Lay the foundation for a habit of continuous improvement
• “First, do no harm.”
  – Insist on strong, generalizable evidence of value before supporting any broad-based revision in curriculum or practice.
For More Information

Please visit the Panel’s site online at:
http://www.ed.gov/MathPanel