

An algebra problem

The following question was given to 8th grade algebra students.
You are simplifying

$$7 - 2(3 - 8x).$$

Which of the expressions is a correct next step?

	Correct	Not correct
$5(3 - 8x)$		
$7 - 2(-5x)$		
$7 - 6 - 16x$		
$7 - 6 + 16x$		

For each expression, explain why you made the choice you did.

Student responses

$$7 - 2(3 - 8x)$$

- $7 - 2(-5x)$ is correct. Always do the parentheses first!
- $7 - 6 + 16x$ is not correct. Where did the 16 come from?
- Can't use the distributive law because of the negative 2.
- $7 - 6 + 16x$ is correct. You have to double multiply first.

What would you say to these students?

Reading and writing expressions

6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.

- a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract y from 5” as $5 - y$.*

Write a numerical expression for the change from a \$10 bill after buying a book at various prices.

Price of book (\$)	5.00	6.49	7.15	y
Change from \$10	$10 - 5$	$10 - 6.49$	$10 - 7.15$	$10 - y$

6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.

- b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.*

6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.

- c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

The evils of PEMDAS

$$n - 2 + 5 = n - 7$$

(correct according to PEMDAS)

$$8 \times (5 + 1) = 8 \times 5 + 8 \times 1 = 40 + 8 = 48$$

(incorrect according to PEMDAS)

- Parentheses indicate how an expression is to be interpreted, not how it is to be calculated.
- What is the correct way to state the convention about order of operations?

The value of leaving expressions unevaluated

Fred and George Weasley make 150 “Deflagration Deluxe” boxes of Weasleys’ Wildfire Whiz-bangs at a cost of 17 Galleons each, and sell them for 20 Galleons each. What is their profit?

Equivalent expressions

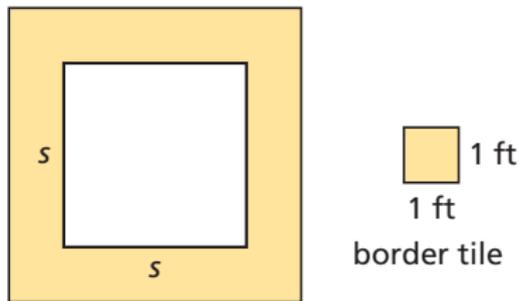
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*

6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

Equivalent expressions

7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”*

Writing expressions in different forms



In expressing the number of tiles needed to border a square pool with side length s feet (where s is a whole number), students might write $4(s + 1)$, $s + s + s + s + 4$, or $2s + 2(s + 2)$, each indicating a different way of breaking up the border in order to perform the calculation. They should see all these expressions as equivalent.

A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Resistors in parallel

$$\text{Total resistance} = \frac{R_1 R_2}{R_1 + R_2}$$

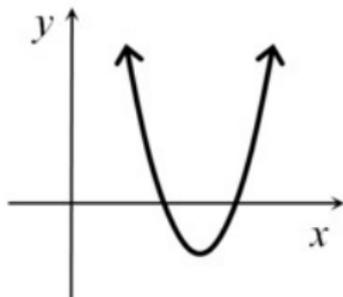
Equivalent form

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{1}{\frac{R_1 + R_2}{R_1 R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- a Factor a quadratic expression to reveal the zeros of the function it defines.
- b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- c Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Which of the following equations could describe the function whose graph is shown below? Explain.



$$f_1(x) = (x + 12)^2 + 4$$

$$f_2(x) = -(x - 2)^2 - 1$$

$$f_3(x) = (x + 18)^2 - 40$$

$$f_4(x) = (x - 10)^2 - 15$$

$$f_5(x) = -4(x + 2)(x + 3)$$

$$f_6(x) = (x + 4)(x - 6)$$

$$f_7(x) = (x - 12)(-x + 18)$$

$$f_8(x) = (20 - x)(30 - x)$$

Solving equations as reasoning

6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Transition from numerical to algebraic solutions

Daniel went to visit his grandmother, who gave him \$5.50. Then he bought a book costing \$9.20. If he has \$2.30 left, how much money did he have before visiting his grandmother?

J. bought three packs of balloons. He opened them and counted 12 balloons. How many balloons are in a pack?

Example: Reasoned solving of equations

Task

A bar of soap on a pan of a scale balances $\frac{3}{4}$ of a bar of soap and $\frac{3}{4}$ of a pound on the other pan. How much does the bar of soap weigh? (From *The Moscow Puzzles*).

In the Classroom (two 9th grade student solutions)

Alice: If x is the weight of the soap in pounds, then one side of the balance weighs x pounds and the other weighs $\frac{3}{4}x + \frac{3}{4}$, so $x = \frac{3}{4}x + \frac{3}{4}$. Subtracting $\frac{3}{4}x$ from both sides $\frac{1}{4}x = \frac{3}{4}$ so $x = 3$.

Barbie: You don't need equations! If a bar of soap balances $\frac{3}{4}$ of a bar and $\frac{3}{4}$ pounds, take $\frac{3}{4}$ of a bar off each side. That leaves $\frac{1}{4}$ of a bar on one side and $\frac{3}{4}$ lb on the other. So a quarter of a bar weighs $\frac{3}{4}$ of a pound. A full bar is four quarters, and that will make 3 lb.